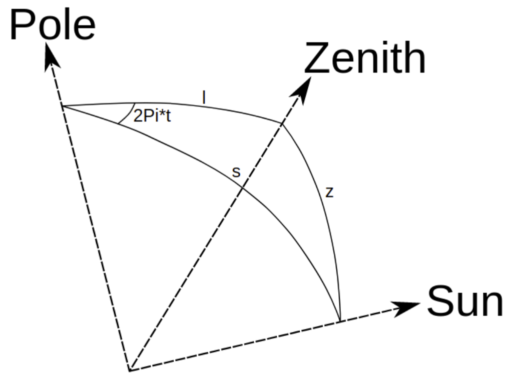

Figure 1: Spherical trigonometry

Approximating the Sun elevation

The Sun elevation as a function of time can easily be approximated from max/min solar elevations, if we only model earth rotation effects. The resulting precision is about 5 minutes.

Solar elevation

From figure 1, using the *cosine rule for sides* from *spherical trigonometry* gives,

$$\begin{aligned} \cos(z) &= \cos(s) * \cos(l) + \sin(s) * \sin(l) * \cos(2\pi t) \\ z &= \frac{\pi}{2} - e \end{aligned}$$

where e is the elevation and the solar time, t, is the fraction of day starting at noon. We assume constant Sun position, s, and observer position, l, which gives us the following relationship between the elevation and solar time,

$$\sin(e) = A + B\cos(2\pi t) \tag{1}$$

where A and B are constant (depend on s and l).

A and B from solar elevation

Let us assume that we know the max and min solar elevation, e_{max} and e_{min}. We may write,

$$\begin{aligned} \sin(e_{max}) &= A + B \\ \sin(e_{min}) &= A - B \\ A &= \frac{\sin(e_{max}) + \sin(e_{min})}{2} \quad (2) \\ B &= \frac{\sin(e_{max}) - \sin(e_{min}))}{2} \quad (3) \end{aligned}$$

Nautical twilight

The time of -6 degree elevation (Nautical twilight start/end) is found using 1, 2 and 3.

$$\begin{aligned} \sin(-6 \cdot \frac{2\pi}{360}) &= A + B\cos(2\pi t) \\ \delta t &= \frac{1}{2\pi} \arccos(\sin\left(\frac{-6 \cdot \frac{2\pi}{360} - A}{B}\right)) \end{aligned}$$

Nautical twilight starts,

$$\begin{aligned} t &= N + \delta t, \\ N &= 0, 1, 2, 3... \end{aligned}$$

Nautical twilight ends

$$\begin{aligned} t &= N - \delta t, \\ N &= 0, 1, 2, 3... \end{aligned}$$